

# Strong field physics in condensed matter\*

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## Abstract

There are deep similarities between non-linear QFT studied in high-energy and non-equilibrium physics in condensed matter. Ideas such as the Schwinger mechanism and the Volkov state are deeply related to non-linear transport and photovoltaic Hall effect in condensed matter. Here, we give a review on these relations.

## INTRODUCTION

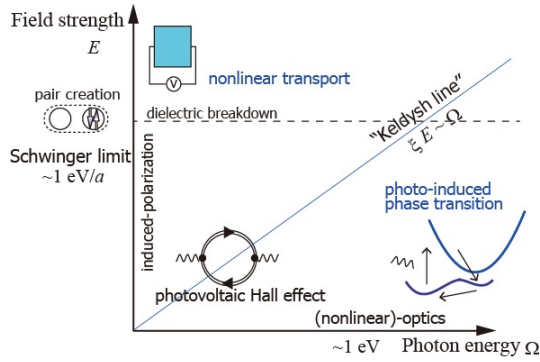


Figure 1: Several phenomena in condensed matter physics in strong electric fields plotted in the  $(E, \Omega)$ -space.

In strong field physics, researchers are interested in the change of the “quantum vacuum” due to strong external fields. A typical example is the decay of the QED vacuum in strong electric fields due to the Schwinger mechanism [1]. When a strong enough electric field is applied to the vacuum, pair creation of electrons and positrons takes place and the insulation break down. The threshold for this phenomenon is known as Schwinger’s critical field and is given by  $E_{\text{th}} = m_e^2/e = 1.3 \times 10^{16} \text{V/cm}$ . Since the critical field is extremely strong, direct experimental verification is still a challenge. On the other hand, in the condensed matter community, there is an increasing interest in non-equilibrium phase transitions and non-linear transport in strongly correlated electron systems (Fig. 1)[2, 3, 4, 5, 6, 7]. In the experiments, one also applies strong electric fields and the original insulating phase is destroyed. However, the threshold for dielectric breakdown is orders of magnitude smaller than the Schwinger mechanism in QED since the excitation gap is far smaller. This makes condensed matter systems to be an idealistic playground to test and develop theoretical ideas in non-linear

QFT. Non-linear physics has been studied rather independently in the two fields, high energy and condensed matter, during the past few decades, and several parallel ideas were developed. The aim of this article is to explain some of the correspondences (Table 1).

## PAIR CREATION IN STRONG ELECTRIC FIELDS

*Heisenberg-Euler’s effective action and the non-linear extension of the Berry’s phase approach to polarization [4]:*

We study lattice electrons in homogeneous electric fields. In the time-dependent gauge, this can be realized by adding a time dependent phase to the hopping term in the lattice Hamiltonian. For example, for a one-dimensional model, a typical Hamiltonian is given by

$$H(\Phi) = - \sum_{i=1}^L \sum_{\sigma} (e^{i\Phi} c_{i+1\sigma}^\dagger c_{i\sigma} + e^{-i\Phi} c_{i\sigma}^\dagger c_{i+1\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_i V_i n_i.$$

We impose periodic boundary condition and the phase  $\Phi$  is proportional to the magnetic flux through the ring ( $L$  number of sites). The time derivative of the magnetic flux is related to the applied electric field through  $F(t) = eaE(t) = d\Phi(t)/dt$ , where  $e$  is the charge quantum and  $a$  the lattice constant.  $U$  represents on-site Coulomb repulsion and  $V_i$  the local potential. The hopping term is set to unity. The Hubbard model ( $U > 0$ ,  $V_i = 0$ ) at half-filling is in the Mott insulating phase for positive  $U$  in one dimension.

Here, we study what happens to the an insulator when we apply strong electric fields. We denote the eigenstates of the Hamiltonian  $H(\Phi)$  by  $|\psi_n(\Phi)\rangle$ ,  $n = 0, 1, \dots$  and study the time evolution starting from the groundstate  $|\psi_0(\Phi)\rangle$ . The groundstate-to-groundstate amplitude defined by

$$\Xi(t) \equiv \langle \psi_0(\Phi(t)) | e^{-i \int_0^t H(\Phi(s)) ds} | \psi_0(0) \rangle e^{i \int_0^t E_0(\Phi(s)) ds} \quad (2)$$

is of central importance. In the long time limit, an asymptotic behavior ( $d$  is dimension)  $\Xi(t) \sim e^{itL^d \mathcal{L}}$  is expected to take place where  $\mathcal{L}$  is the condensed matter version of the *Heisenberg-Euler effective Lagrangian*. The imaginary part describes quantum tunneling where  $\Gamma(F)/L^d \equiv 2\text{Im } \mathcal{L}(F)$  gives the speed of the exponential decay of the vacuum (groundstate). This quantity is proportional to the decay rate of the Loschmidt Echo  $L(t) = |\Xi(t)|^2$ . The real part  $\text{Re} \mathcal{L}$  is written in terms of a non-adiabatic phase

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Table 1: Related ideas in strong field physics

| High Energy                                     | Condensed Matter   |
|---|--|
| Schwinger mechanism in QED                      | Landau-Zener tunneling in band insulators                                |
| Heisenberg-Euler effective Lagrangian           | Non-adiabatic geometric phase, Loschmidt Echo                            |
| Vacuum polarization                             | Extended Berry's phase theory of polarization                            |
| Pair creation in interacting systems (e.g. QCD) | Many-body Schwinger-Landau-Zener mechanism in strongly correlated system |
| Dirac particles in circularly polarized light   | Photovoltaic Hall effect   |
| Furry picture                                   | Floquet picture  |

called the Aharonov-Anandan phase (which we denote  $\gamma$ ) that the wave function acquires during the time-evolution. For band insulators ( $U = 0$ ) in dc-electric fields, the effective Lagrangian becomes [4]

$$\text{Re } \mathcal{L}(F) = -F \int_{\text{BZ}} \frac{d\mathbf{k}}{(2\pi)^d} \frac{\gamma(\mathbf{k})}{2\pi}, \quad (3)$$

$$\text{Im } \mathcal{L}(F) = -F \int_{\text{BZ}} \frac{d\mathbf{k}}{(2\pi)^d} \frac{1}{4\pi} \ln[1 - p(\mathbf{k})], \quad (4)$$

where the momentum integral is over the Brillouin Zone (BZ). There is an interesting parallel theory developed in the condensed matter community. This is known as the Berry's phase theory of polarization[8, 9, 10, 11, 12], where the ground-state expectation value of the twist operator  $e^{-i\frac{2\pi}{L}\hat{X}}$ , which shifts the phase of electron wave functions on site  $j$  by  $-\frac{2\pi}{L}j$ , plays a crucial role. It was revealed that the real part of a quantity

$$w = \frac{-i}{2\pi} \ln \langle 0 | e^{-i\frac{2\pi}{L}\hat{X}} | 0 \rangle \quad (5)$$

gives the electric polarization  $P_{\text{el}} = -\text{Re} w$  [10] while its imaginary part gives a criterion for metal-insulator transition, i.e.,  $D = 4\pi \text{Im} w$  is finite in insulators and divergent in metals [11]. The present effective Lagrangian can be regarded as a non-adiabatic (finite electric field) extension of  $w$ . To give a more accurate argument, we rewrite the effective Lagrangian in the time-independent gauge

$$\mathcal{L}(F) \sim \frac{-i\hbar}{\tau L} \ln \left( \langle 0 | e^{-\frac{i}{\hbar}\tau(H+F\hat{X})} | 0 \rangle e^{\frac{i}{\hbar}\tau E_0} \right) \quad (6)$$

for  $d = 1$ . Let us set  $\tau = \hbar/LF$  and consider the small  $F$  limit. For insulators we can replace  $H$  with the groundstate energy  $E_0$  to have  $\mathcal{L}(F) \sim wF$  in the linear-response regime. Thus the real part of Heisenberg-Euler's expression[13] for the non-linear polarization  $P(F) = -\partial \mathcal{L}(F)/\partial F$  naturally reduces to the Berry's phase formula  $P_{\text{el}}$  in the small field limit  $F \rightarrow 0$ . Its imaginary part gives the criterion for photo-induced metal-insulator transition, originally proposed for the zero field case.

### Many-body Schwinger-Landau-Zener mechanism in strongly correlated insulators:

Next, let us consider dielectric breakdown in a strongly correlated system. In the one-dimensional Mott insulator

where the groundstate is a state with one electron per site, the relevant charge excitations are doublons, i.e., doubly occupied sites, and holes, i.e., sites with no electron. Pairs of doublons and holes play a similar role as the pair of electrons and positrons in the Schwinger mechanism. Indeed, it has been shown that dielectric breakdown in Mott insulators takes place due to pair production of charge excitations through quantum tunneling, which is called the

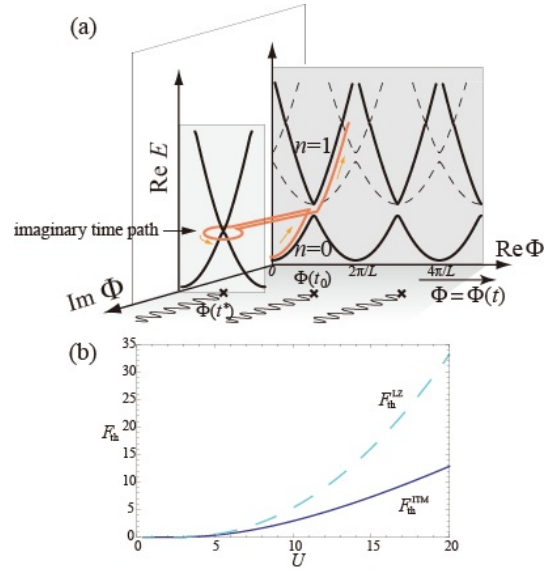


Figure 2: (a) Many-body energy levels against the complex AB flux  $\Phi$  for a finite, half-filled 1D Hubbard model ( $L = 10$ ,  $N_{\uparrow} = N_{\downarrow} = 5$ ,  $U = 0.5$ ). Only charge excitations are plotted. Quantum tunneling occurs between the groundstate (labeled as  $n = 0$ ) and a low-lying excited state ( $n = 1$ ) as the flux  $\Phi(t) = Ft$  increases on the real axis, while the tunneling is absent for the states plotted as dashed lines. The wavy lines starting from the singular points ( $\times$ ) at  $\Phi(t^*)$  represent the branch cuts for different Riemann surfaces, along which the solutions  $n = 0$  and  $n = 1$  are connected. In the DDP approach, the tunneling factor is calculated from the dynamical phase associated with adiabatic time evolution (DDP path) that encircles a gap-closing point at  $\Phi(t^*)$  on the complex  $\Phi$  plane. (b) Threshold electric field obtained by the imaginary time method (solid) and the naive Landau-Zener formula (dashed).

many-body Schwinger-Landau-Zener mechanism [3, 4, 5]. The tunneling probability for the lowest excited state in the one-dimensional Hubbard model was recently calculated by combining the imaginary time method with the exact Bethe ansatz solution [5].

The imaginary time method has been widely used in quantum physics, e.g., in quantum chemistry [14, 15] and in high energy [16, 17, 18]. In high energy, the tunneling rate for Dirac particles in ac-electric fields was calculated by analytically extending the time-evolution to complex time processes. This method can be readily applied to the one-dimensional Hubbard model since the wave functions, including the excited states, are analytically known. Using Bethe ansatz, these states can be described as a linear combination of plane waves with the momentum (rapidity) determined by the Lieb-Wu equation [19]. In the infinite size limit, the Lieb-Wu equation can be solved even in the presence of a complex phase, i.e.,  $\Phi \rightarrow \Phi + i\Psi$  [20, 5]. In the half-filled Hubbard model, the lowest energy excited state is given by the string solution [21, 22, 23, 24]. We denote the energy difference between this state and the ground-state by  $\Delta = E_1 - E_0$ . For dc-electric fields, quantum tunneling to the lowest energy state dominates and one can express the decay rate by

$$\Gamma/L \sim -\frac{\alpha F}{2\pi} \ln[1 - \exp(-\pi F_{\text{th}}/F)], \quad (7)$$

where  $F_{\text{th}}$  is the threshold field for this process and  $\alpha$  a numerical factor. In the imaginary time method, an adiabatic evolution to a gap closing point in the complex  $\Phi + i\Psi$  space is performed and the tunneling probability depends on the dynamical phase. The gap closing in complex  $\Phi$  has been studied in ref. [20], while in ref. [25] a related problem was studied in order to calculate the localization length in a Mott insulator. The Bethe ansatz solution for the Hubbard model with finite  $\Psi$  is represented by a rapidity distribution with end points  $\pm a + ib$ . The imaginary part  $b$  is related to  $\Psi$  by a condition that the charge quantum number should remain real. The threshold for the tunneling is given by[5]

$$\begin{aligned} F_{\text{th}}^{\text{ITM}} &= \frac{2}{\pi} \int_0^{b_{\text{cr}}} \Delta \frac{d\Psi}{db} db \\ &= \frac{2}{\pi} \int_0^{\sinh^{-1} u} 4 \left[ u - \cosh b + \int_{-\infty}^{\infty} d\omega \frac{e^{\omega \sinh b} J_1(\omega)}{\omega(1 + e^{2u|\omega|})} \right] \\ &\quad \times \left[ 1 - 2 \cosh b \int_0^{\infty} d\omega \frac{J_0(\omega) \cosh(\omega \sinh b)}{1 + e^{2u\omega}} \right] db. \end{aligned} \quad (8)$$

Its  $U$ -dependence is plotted in Fig. 2 (b) (solid line). This indicates a collective nature of the breakdown (i.e., the threshold much smaller than a naive  $U$ ). In other words, the tunneling takes place not between neighboring sites, but over an extended region due to a *leakage of the many-body wave function*, where the size is roughly given by the localization length [25]. One can compare this result with the naive estimate for the tunneling threshold obtained by the Landau-Zener formula [3, 4]

$$F_{\text{th}}^{\text{LZ}} = \frac{(\Delta/2)^2}{v}, \quad (9)$$

where  $\Delta$  is the charge gap (Mott gap) [26], and  $v$  is the slope of the adiabatic levels ( $v \sim 2$  when  $U$  is small and the system size is small). From Fig. 2 (b), one notice that the result of the imaginary time method and the Landau-Zener formula coincides in the small  $U$  limit. However, when the interaction becomes stronger, large deviation takes place, with different asymptotic behaviors  $F_{\text{th}}^{\text{ITM}} \propto U - \text{const.}$  and  $F_{\text{th}}^{\text{LZ}} \propto U^2$ . Dielectric breakdown in the Hubbard model has been studied numerically and threshold behaviors in the decay rate [4] as well as in the current[27, 6] are observed. Further research to explore the strong  $U$  limit is yet to be done.

In ac-electric fields, the tunneling behavior can still be realized. The Keldysh line in Fig. 1 corresponds to the line where the adiabaticity parameter [28]  $\tilde{\gamma} = \Omega/F\xi$  crosses unity ( $\xi$  is the localization length in the present case). For  $\tilde{\gamma} \gg 1$ , the tunneling process is the dominant cause of carrier production, while when  $\tilde{\gamma} \ll 1$ , multiphoton absorption wins. In the next section, another interesting effect of the ac-electric field is discussed where light can be used to control the topology of the band structure.

## PHOTOVOLTAIC HALL EFFECT

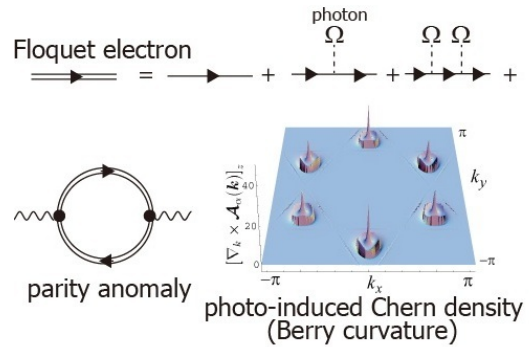


Figure 3: Fig.5 (upper) Greenfs function in the Floquet picture. (lower left) Diagram for conductivity. (lower right) Photo-induced Berry curvature (Chern density) of graphene in circularly polarized light.

Dirac electrons are now becoming one of the central topics in condensed matter after its experimental realization in graphene[29]. Electrons in strong background electric fields is an important problem and can be experimentally studied by optical and transport methods. In non-linear QED, this problem has been studied by Volkov for ac-fields where he found that electron wave functions acquire a non-trivial phase[30]. However, in condensed matter, the electrons subject to strong lasers behave differently from the Volkov state of relativistic Dirac particles. This is because the velocity, or the “speed of light”, of Dirac electrons in condensed matter systems are much slower compared to the actual speed of light and the ac-field breaks the “Lorentz symmetry” of the electron. This leads to a dynamical gap opening in the pseudo-energy diagram [31, 7].

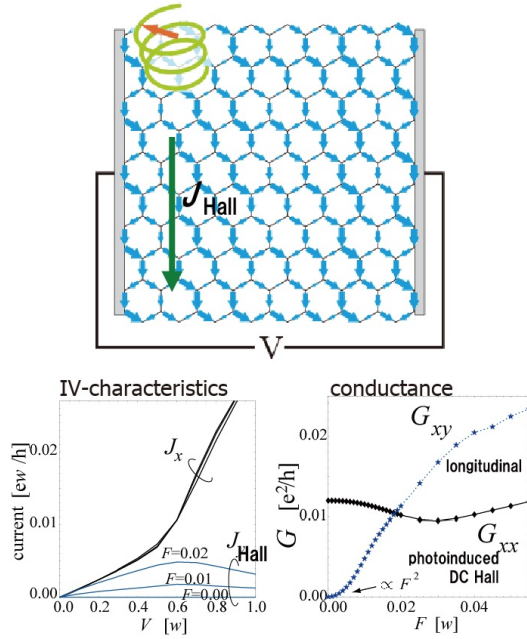


Figure 4: (upper) DC current. (lower left) IV-characteristics. (lower right) Conductance.

Especially, in a circularly polarized light, a gap opens at the Dirac point [7]. This has an important physical consequence since a gap of a 2+1 dimensional Dirac electron is related to parity anomaly and is detectable through transport measurements, i.e., the Hall effect. In 2+1 dimension, the Hall conductivity can be written as a momentum integral of the Berry curvature ( $\sim$  Chern density) over the Brillouin zone. This is known as the TKNN formula[32], and is now extended to ac-driven transport via the Floquet picture ( $\sim$  Furry picture) [7]

$$\sigma_{xy}(\mathcal{A}_{\text{ac}}) = e^2 \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha} f_{\alpha}(\mathbf{k}) [\nabla_{\mathbf{k}} \times \mathcal{A}_{\alpha}(\mathbf{k})]_z. \quad (10)$$

Here,  $\mathcal{A}_{\alpha}(\mathbf{k}) \equiv -i\langle\langle\Phi_{\alpha}(\mathbf{k})|\nabla_{\mathbf{k}}|\Phi_{\alpha}(\mathbf{k})\rangle\rangle$  is the photo-induced artificial gauge field. In the Floquet picture, the Green's function incorporates the effect of photon absorption and emission (Fig. 3 (a)), and Hall conductivity is given by the bubble diagram in the non-interacting case, which is nothing but the parity anomaly diagram. The photo-induced Berry curvature shown in Fig. 3 (c) acts as an artificial magnetic field and becomes finite when the circularly polarized light is introduced.

The current in the presence of circularly polarized light in a graphene ribbon attached to two electrodes is plotted in Fig. 4. The calculation has been done by combining the Keldysh green's function method with the Floquet picture. The Hall current, which is originally absent, increases as the strength of light becomes stronger. The numerical result supports our understanding of the photovoltaic Hall effect obtain by the extended TKNN formula (eqn.(10)).

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